

# Exclusive Central Hadron Production in $p\bar{p}$ Collisions at the Tevatron for $\sqrt{s} = 1960\text{GeV}, 900\text{GeV}$

## Partial Wave Analysis - Update

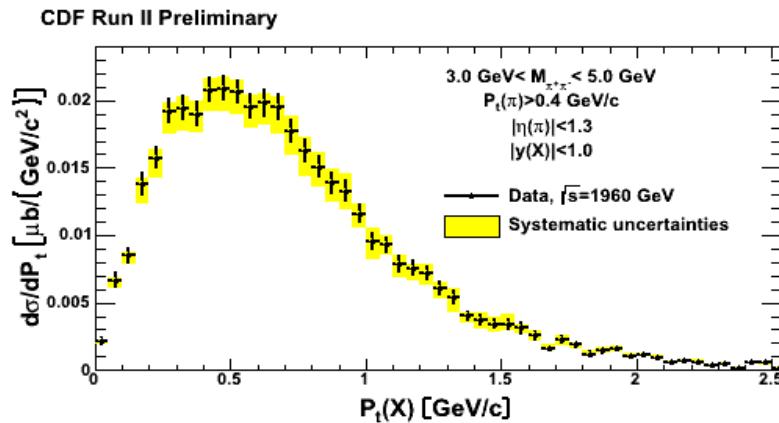
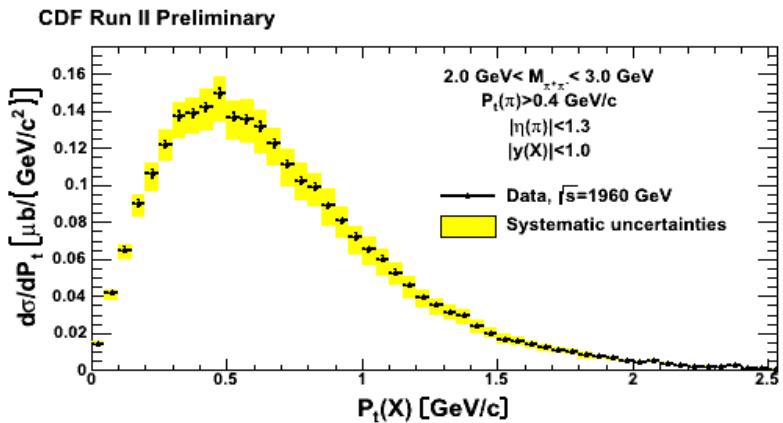
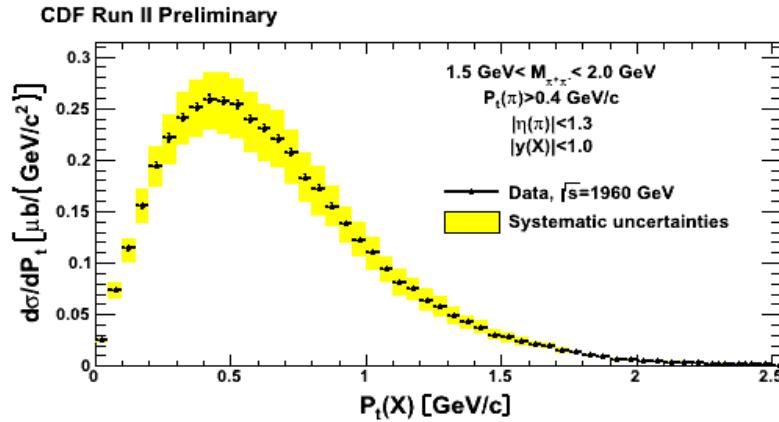
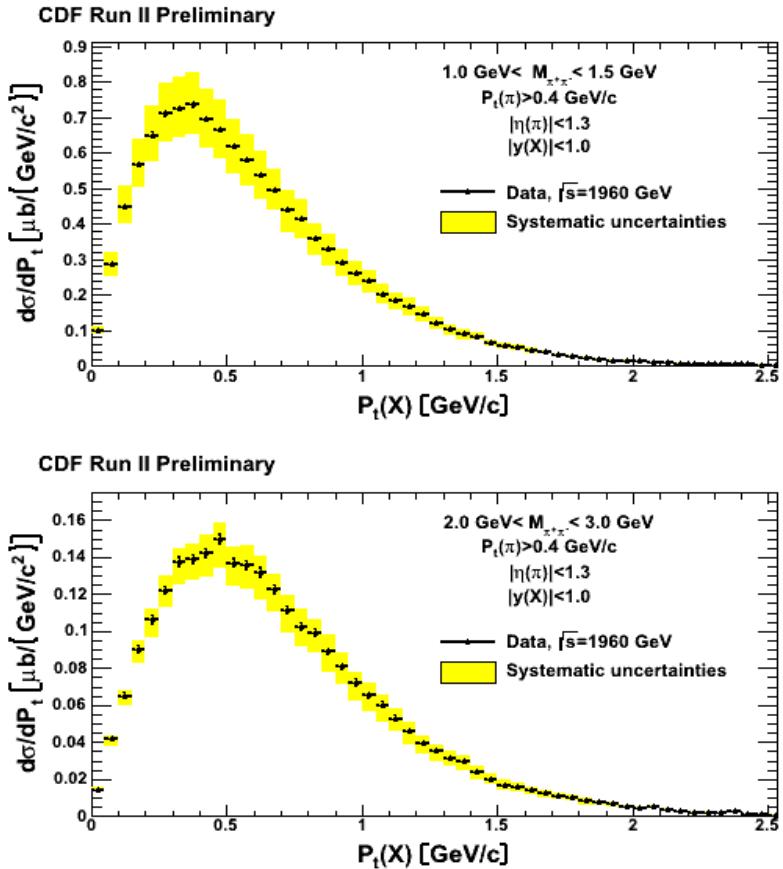
M. Żurek, A. Świech  
Jagiellonian University, Kraków/  
University of Cologne

D. Lontkovskyi, I. Makarenko  
University of Kyiv

M. Albrow, J.S. Wilson  
FNAL, University of Michigan

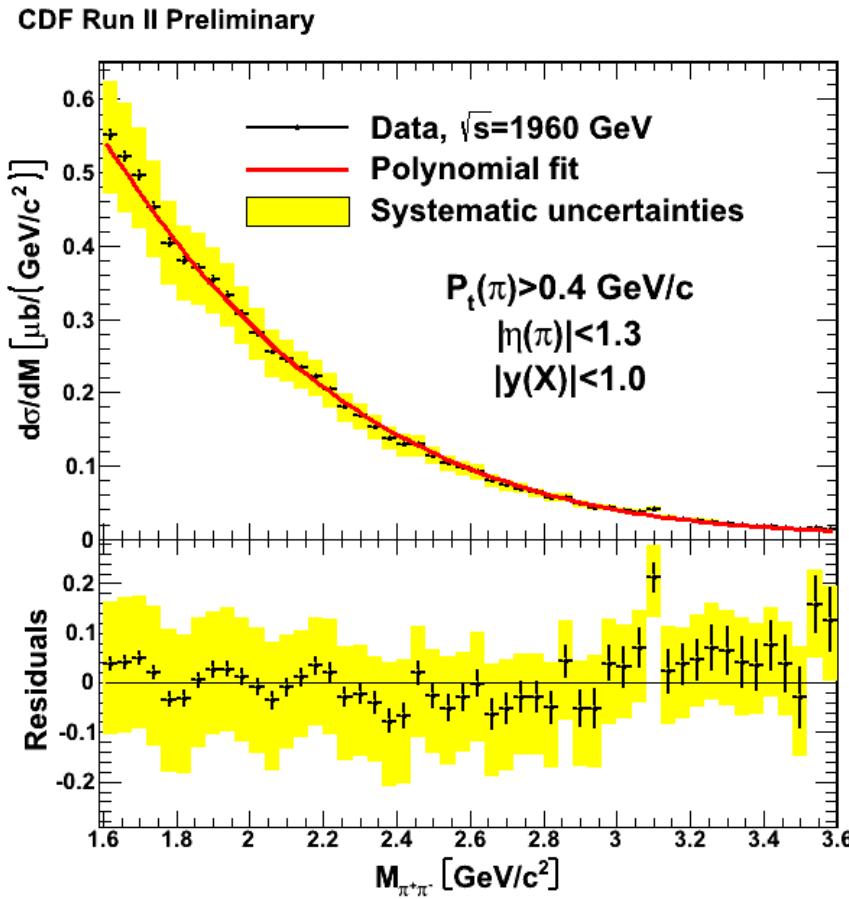


# Pt distribution for different mass ranges





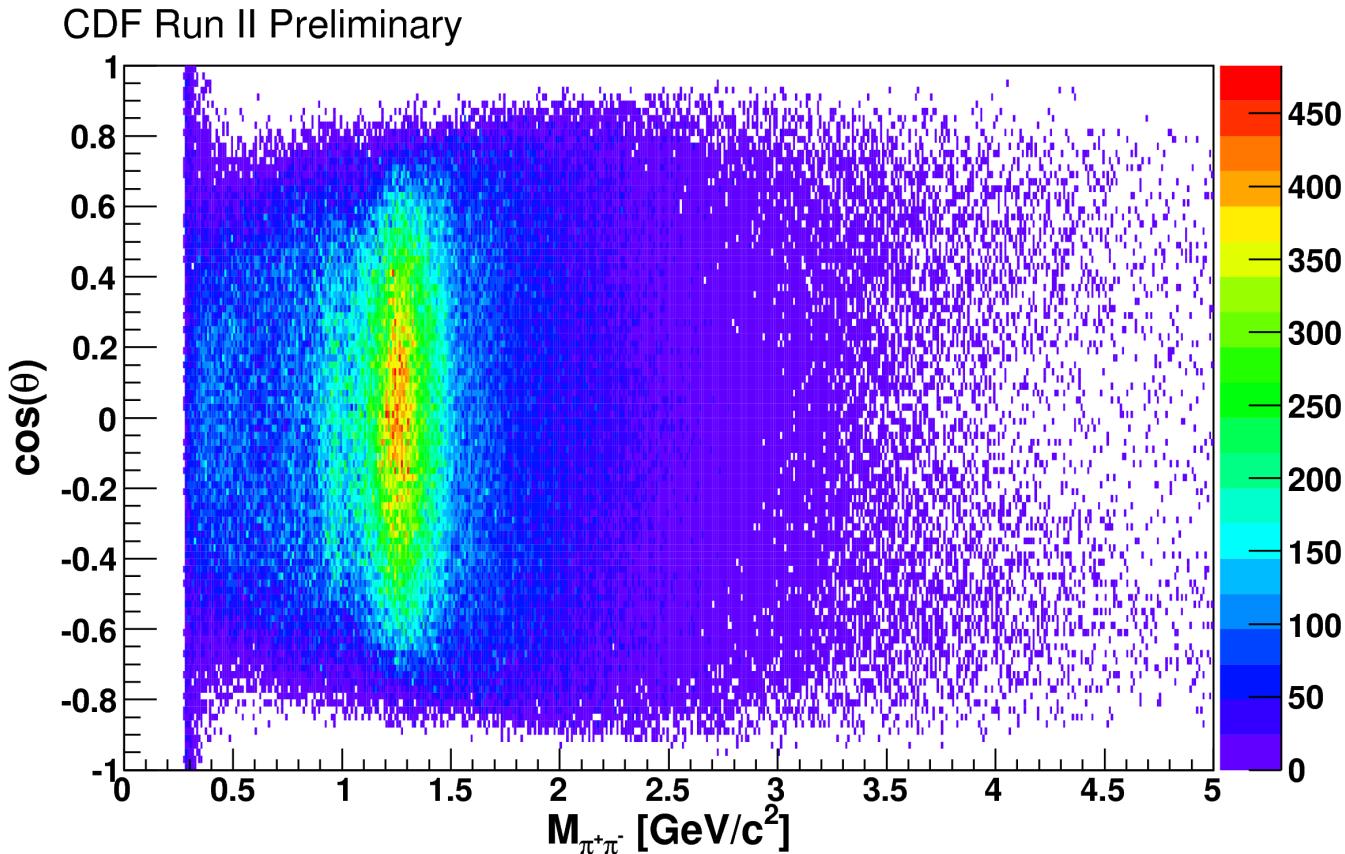
# Mass distribution – tail fit



# PWA

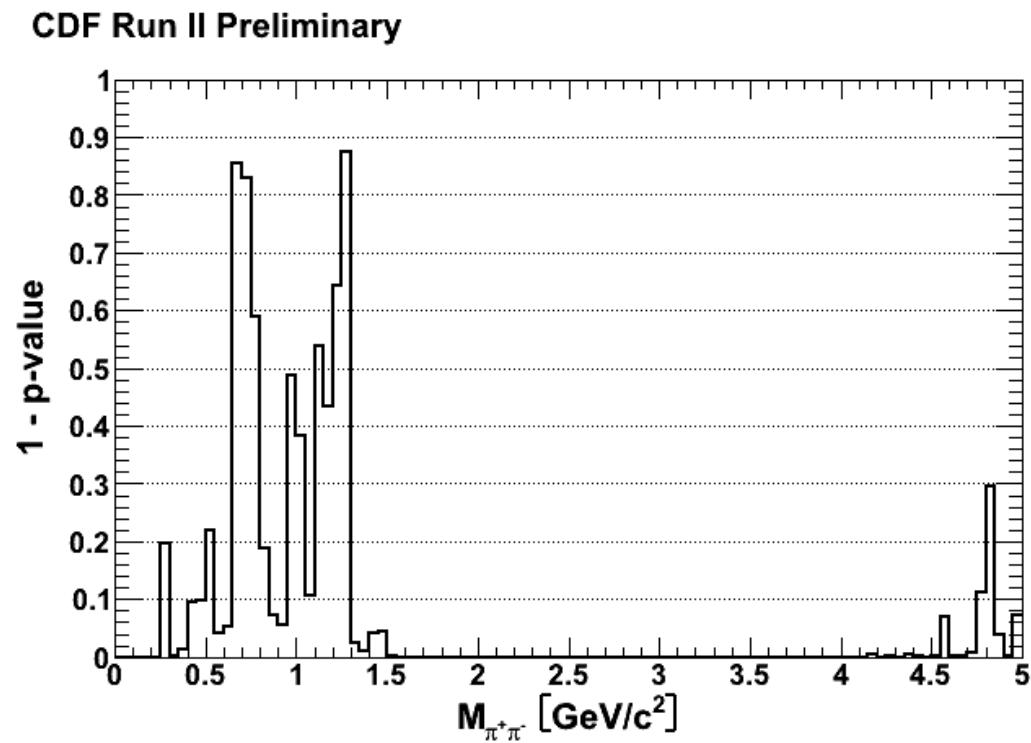
- Comparison of data/MC s-wave  $\cos(\theta)$  distributions
- $H_0$ :  $\cos(\theta)$  distribuants for data and s-wave MC are the same (in mass bins)
- $H_1$ : not  $H_0$ .
- Test type: Smirnov
- Test statistics:  $\lambda$  Kolmogorov

# $\cos(\theta)$ vs Inv Mass



# 1 - p-value distribution

- one means compatible



## Unpolarized cross-section

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2s_a + 1)(2s_b + 1)p^2} \sum_{(\lambda), J, J'} \left( J + \frac{1}{2} \right) \left( J' + \frac{1}{2} \right) (-1)^{\lambda - \mu}$$
$$\cdot \langle \lambda_a \lambda_b | T_J(E) | \lambda_c \lambda_d \rangle^* \langle \lambda_a \lambda_b | T_{J'}(E) | \lambda_c \lambda_d \rangle$$
$$\cdot \sum_{\ell} C(JJ'\ell; \lambda, -\lambda) C(JJ'\ell; \mu, -\mu) P_\ell(\cos \theta)$$

M.Jacob, G.C.Wick, On the general theory of collisions for particles with spin, Ann. Phys. 7, (1959) 404-428.



- ▶  $s_a, s_b$  - spins
- ▶  $J, J'$  - total angular momenta
- ▶  $\lambda_a, \lambda_b, \lambda_c, \lambda_d$  - helicities;  $\mu = \lambda_c - \lambda_d, \lambda = \lambda_a - \lambda_b$
- ▶  $p$  - momentum of initial state particle,  $E$  - c.m. energy
- ▶  $T = i(1 - S)$ ,  $S$  - scattering matrix
- ▶  $C(JJ'\ell; \lambda, -\lambda)$  - C-G coefficients

# Double Pomeron Exchange

Goal:  $\langle \lambda_a \lambda_b | T_J(E) | \lambda_c \lambda_d \rangle = ?$

DPE properties:

- ▶  $\pi^+ \pi^-$  production only via *s*-channel diagrams
- ▶  $0^{++}, 2^{++}, 4^{++}, \dots$  intermediate states only
  - each such state has a definite  $J$
  - $0^{++}$  states contribute only to  $T_0$
- ▶  $s_\pi = 0, s_{\mathbb{P}} = 0, \lambda_\pi, \lambda_{\mathbb{P}} = 0$
- ▶ Therefore:  $\langle \lambda_a \lambda_b | T_J(E) | \lambda_c \lambda_d \rangle$  is a single complex number  $R_J(E)e^{i\phi_J(E)}$

Tool: Measurement of coefficients of Legendre polynomials  $a_\ell$

## $0^{++}$ and $2^{++}$ central state assumption

- ▶  $J, J' = 0, 2 \rightarrow \ell = 0, 2, 4.$
- ▶ Only non-zero C-G coefficients:  $C(000; 00), C(022; 00), C(220; 00), C(222; 00), C(224; 00)$

1.  $\ell = 4 \rightarrow$  only  $J = J' = 2:$

$$a_4 = \left(\frac{9}{7}\right)^2 p^{-2} R_2^2$$

2.  $\ell = 0 \rightarrow J = J' = 2$  or  $J = J' = 0:$

$$a_0 = \frac{1}{4} p^{-2} (R_0^2 + R_2^2)$$

3.  $\ell = 2 \rightarrow J = J' = 2$  or  $J = 0, J' = 2$  or  $J = 2, J' = 0:$

$$a_2 = p^{-2} \left( \frac{5}{2} R_0 R_2 \cos(\phi_2 - \phi_0) + \left(\frac{5}{7}\right)^2 R_2^2 \right),$$

where:  $\delta = \phi_2 - \phi_0$  - relative phase

Finally:

1.  $R_2^2 = p^2 \left(\frac{7}{9}\right)^2 a_4$

2.  $R_0^2 = p^2 \left(4a_0 - \left(\frac{7}{9}\right)^2 a_4\right)$

3.  $\delta = \frac{1}{2} \frac{a_2 - \left(\frac{5}{9}\right)^2 a_4}{\sqrt{\left(\frac{7}{9}\right)^2 a_4 \left(4a_0 - \left(\frac{7}{9}\right)^2 a_4\right)}}$

## Legendre moments - correction for acceptance

Corrected Legendre moments  $\mathbf{b}$  (vector):

$$\mathbf{b} = \mathbf{K}^{-1} \mathbf{a}.$$

$\mathbf{a}$  - not corrected Legendre moments:

$$a_\ell = \frac{\sum_i w_i P_\ell(\cos \theta_i)}{\sum_i w_i},$$

$\mathbf{K}$  following matrix:

$$K_{\ell\ell'} = \frac{\sum_i w_i P_\ell(\cos \theta_i) P_{\ell'}(\cos \theta_i)}{\sum_i w_i},$$

where:  $w_i = w_i^{MCweight} \cdot w_i^{Acc}$ .

<http://www-cdf.fnal.gov/~jsw/internal/GXG/PWA-corrections.md.html>

# Legendre moments - correction for acceptance

## 1. Statistical uncertainties:

$$\text{cov}(b_\ell, b_{\ell'}) = K_{\ell\ell'}^{-1} \text{cov}(a_\ell, a_{\ell'}) \left( K_{\ell\ell'}^{-1} \right)^T$$

We need the covariance of the mean value of the sample.

$$\begin{aligned}\text{cov}(a_\ell, a_{\ell'}) &= \frac{\sum_{ij} w_i w_j \text{cov}(P_\ell(\cos \theta_i), P_{\ell'}(\cos \theta_j))}{\sum_{ij} w_i w_j} \\ &= \frac{\sum_i w_i^2}{\sum_{ij} w_i w_j} \text{cov}(P_\ell(\cos \theta), P_{\ell'}(\cos \theta))\end{aligned}$$

Let us denote:  $V_1 = \sum_i w_i$ ,  $V_2 = \sum_i w_i^2$ , then:

$$\begin{aligned}\text{cov}(a_\ell, a_{\ell'}) &= \frac{V_2}{V_1^2} \frac{V_1}{V_1^2 - V_2} \sum_i w_i (P_\ell(\cos \theta_i) - a_\ell)(P_{\ell'}(\cos \theta_i) - a_{\ell'}) \\ &= \frac{V_2}{V_1^2 - V_2} \left( \frac{\sum_i w_i P_\ell P_{\ell'}}{V_1} - a_\ell a_{\ell'} \right) = \frac{V_2}{V_1^2 - V_2} (\langle P_\ell P_{\ell'} \rangle - a_\ell a_{\ell'})\end{aligned}$$

## Legendre moments - correction for acceptance

**2. Uncertainties linked with  $K^{-1}$  matrix:** related to statistics of our MC sample

M. Lefebvre, R.K. Keeler, R. Sobie, J. White, Propagation of Errors for Matrix Inversion, [arXiv:hep-ex/9909031]

Let us denote:  $\epsilon_{lm} = \langle P_l P_m \rangle$ :

$$\text{cov}(\epsilon_{ab}^{-1}, \epsilon_{cd}^{-1}) = \sum_{ijkl} \epsilon_{ai}^{-1} \epsilon_{jb}^{-1} \epsilon_{ck}^{-1} \epsilon_{ld}^{-1} \text{cov}(\epsilon_{ij}, \epsilon_{kl}),$$

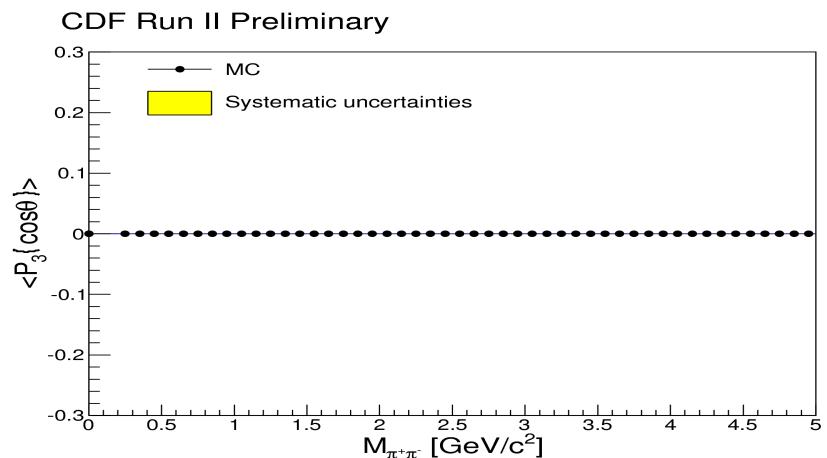
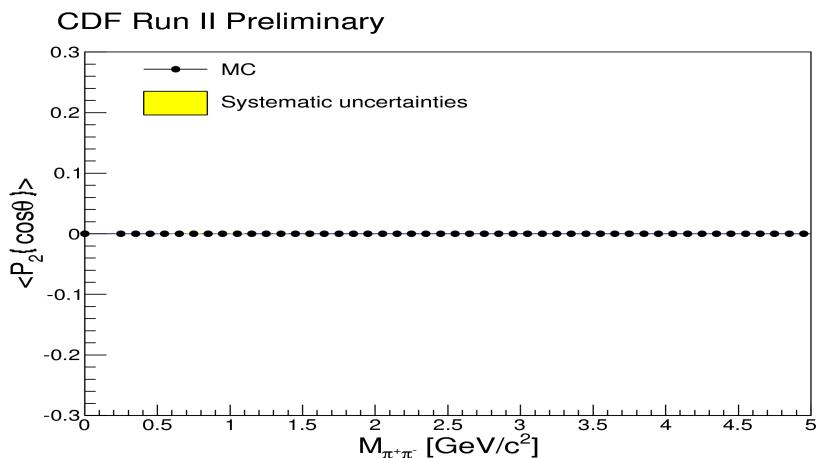
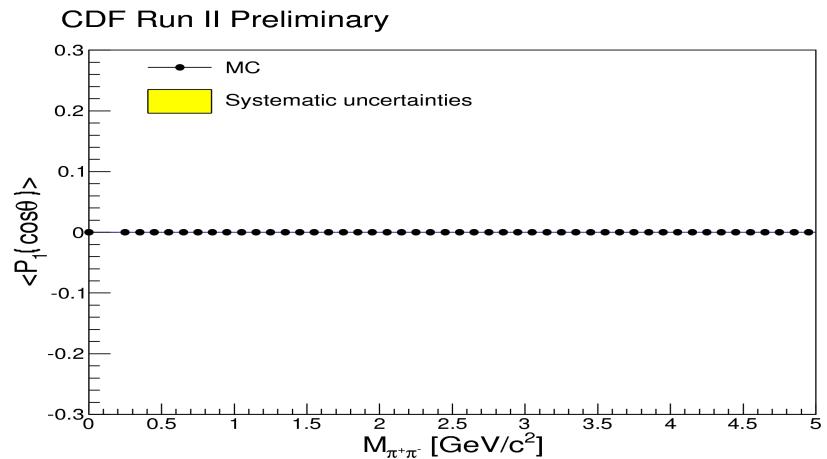
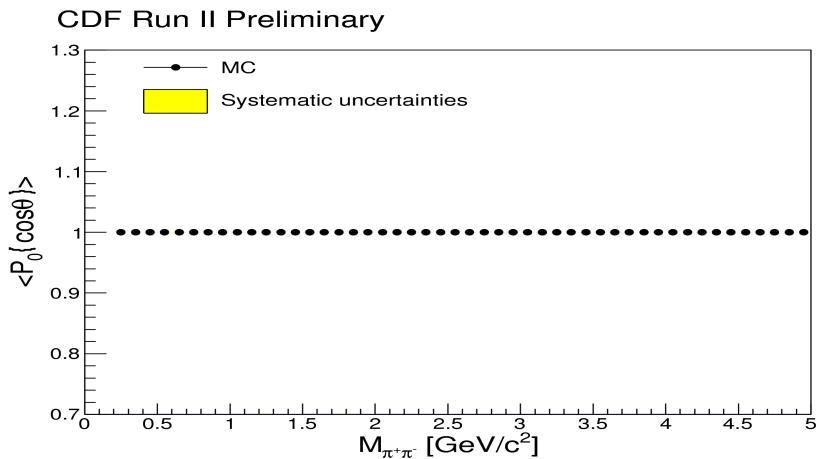
thus:

$$\delta b_i^2 = \sum_{jk} a_j \text{cov}(\epsilon_{ij}^{-1}, \epsilon_{ik}^{-1}) a_k$$

$\text{cov}(\epsilon_{ij}, \epsilon_{kl})$  - calculated in analogous way as in 1.

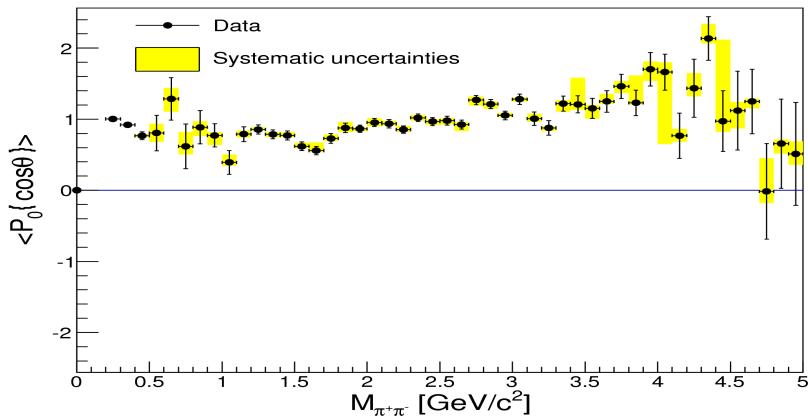
**3. Systematical uncertainties:** We varied all parameters (in Data and MC) and checked the result in Legendre moments plots.

# MC – no weighting

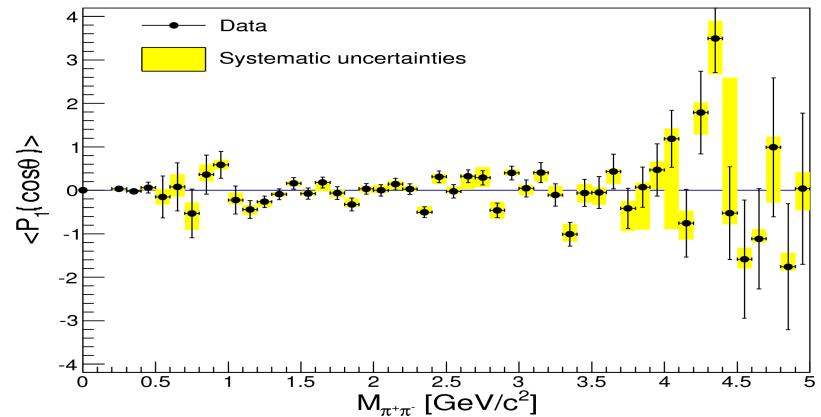


# Data – no MC weighting

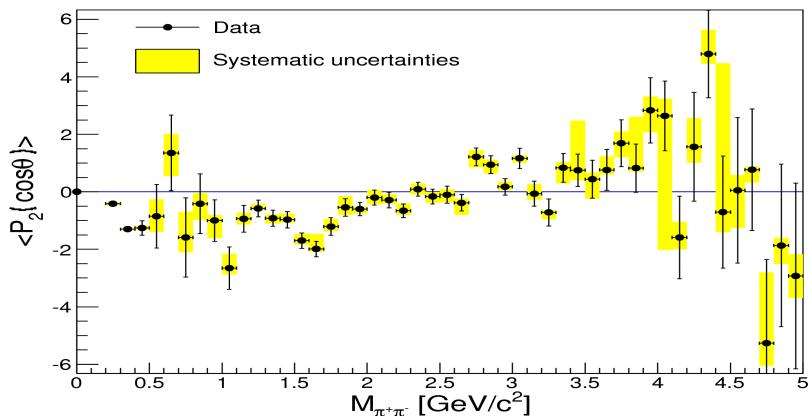
CDF Run II Preliminary



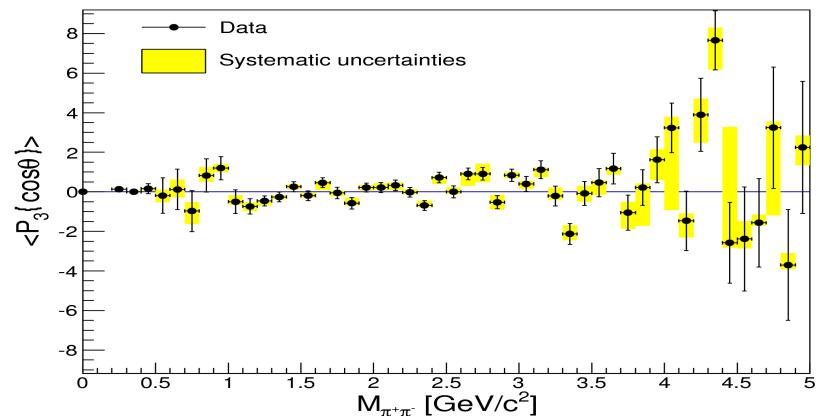
CDF Run II Preliminary



CDF Run II Preliminary

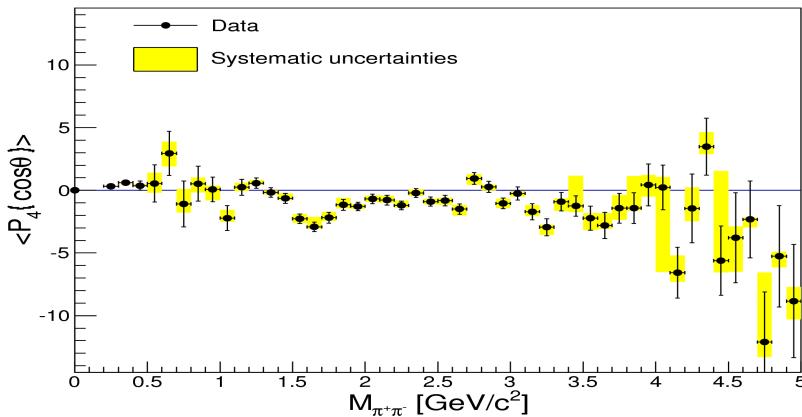


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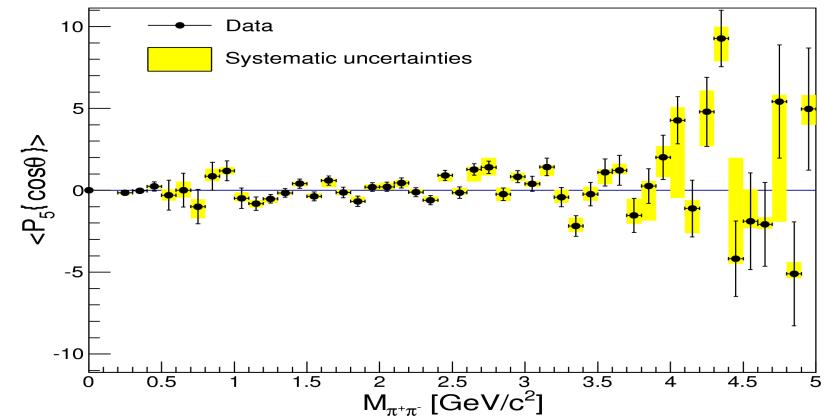


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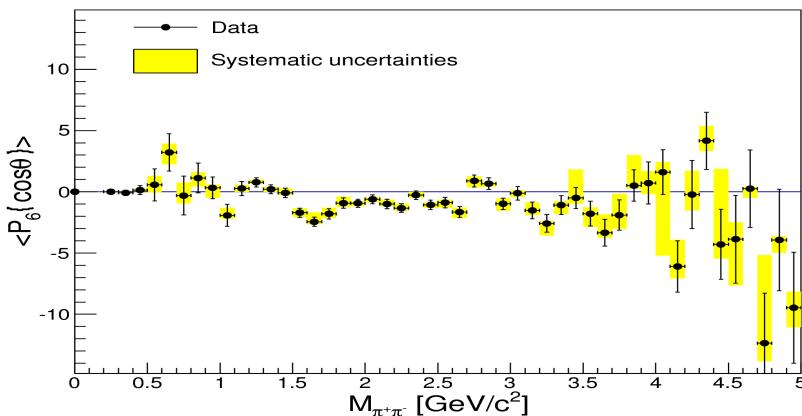
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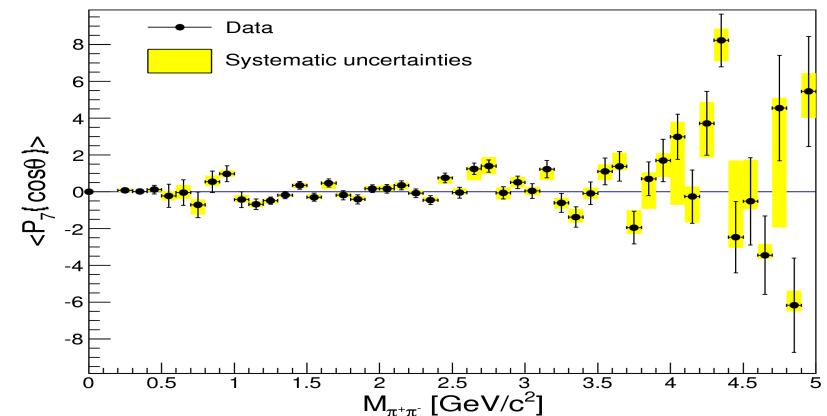
CDF Run II Preliminary



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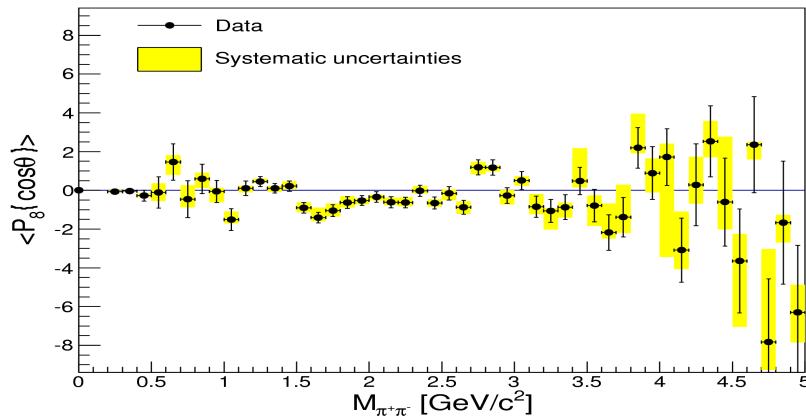


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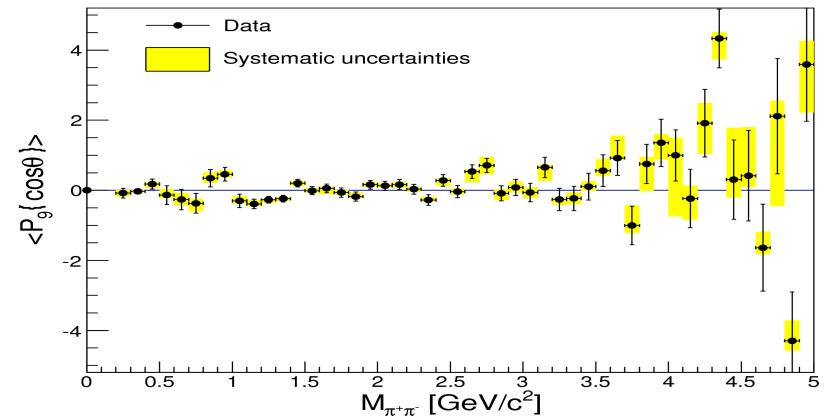


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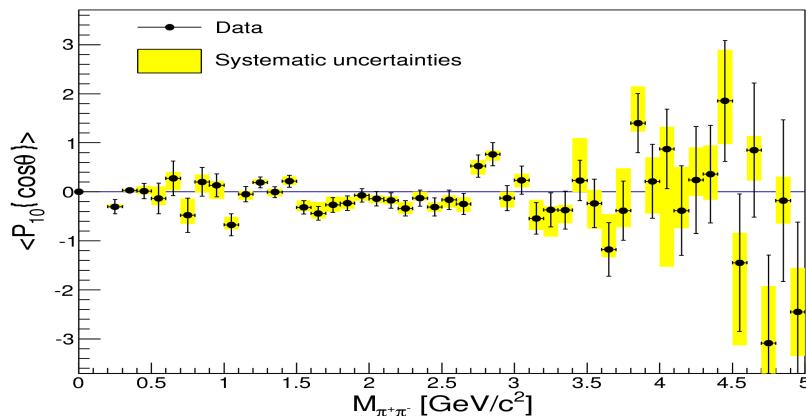
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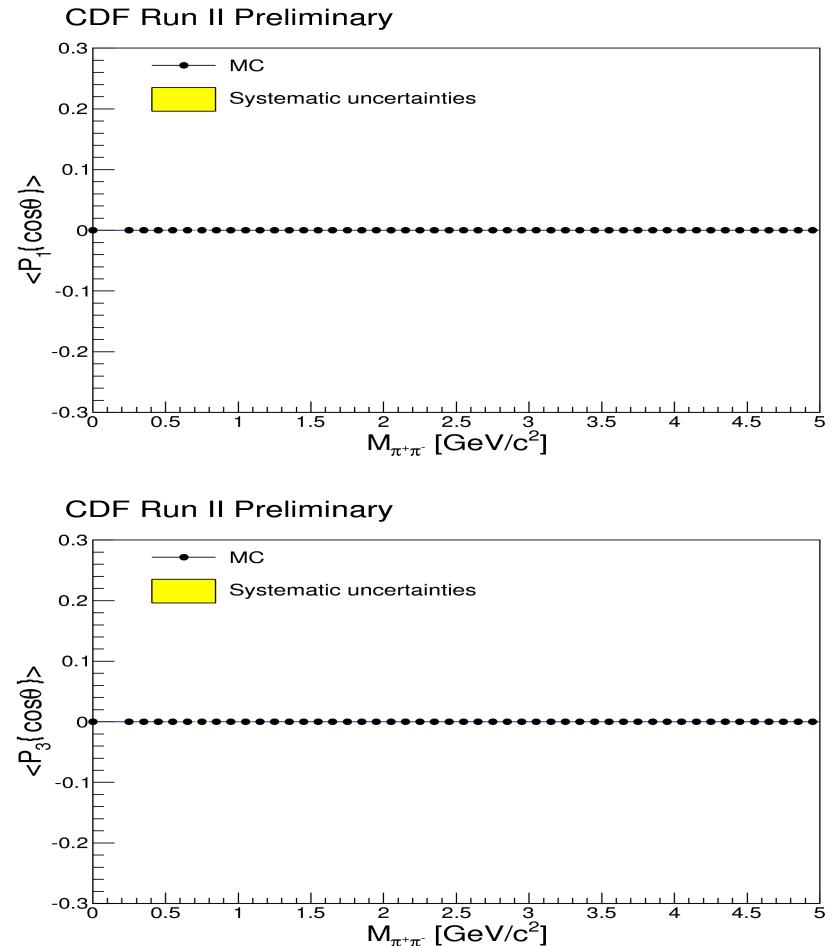
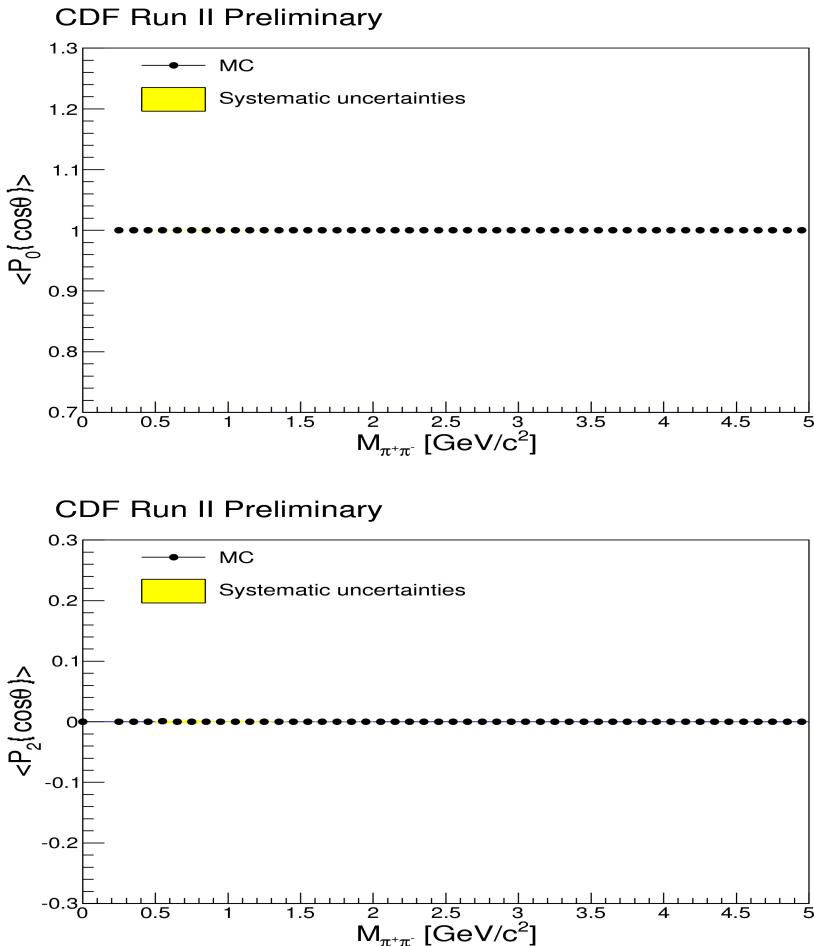
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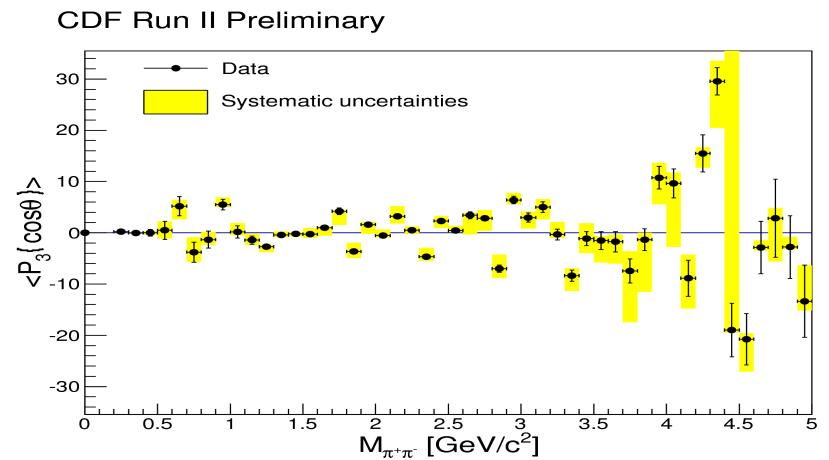
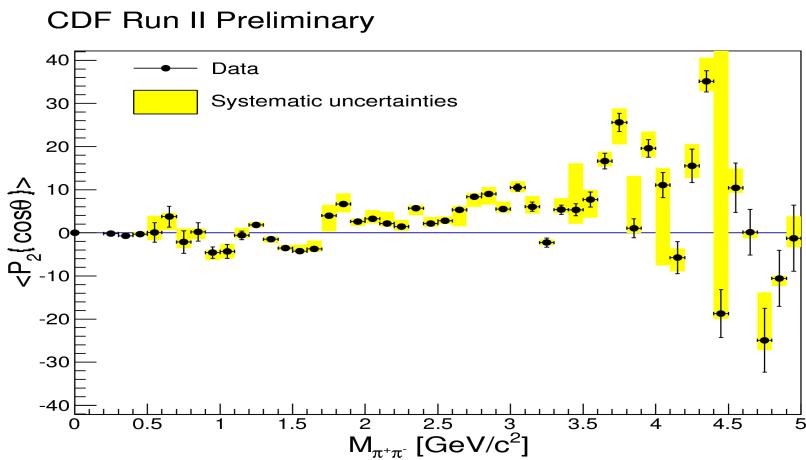
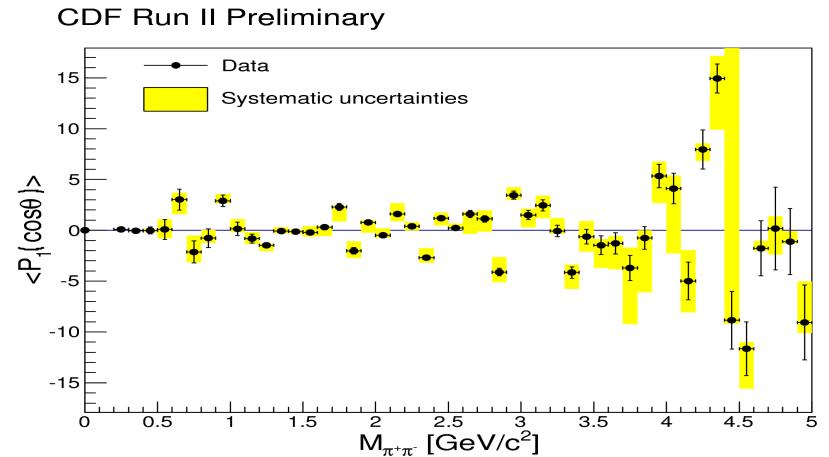
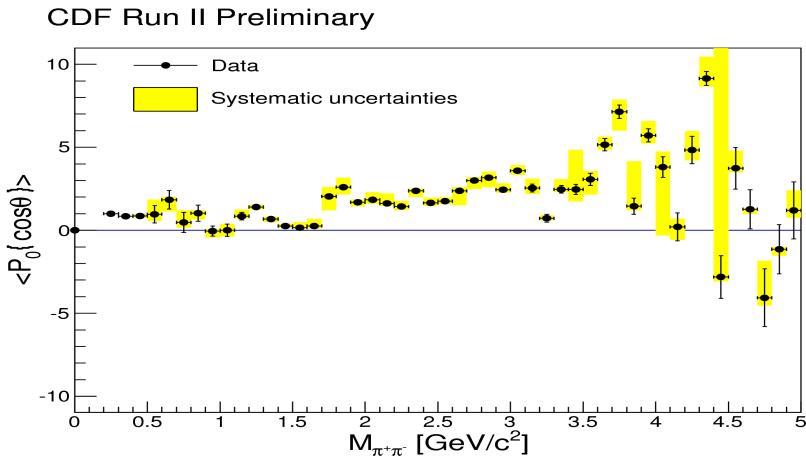
CDF Run II Preliminary



# MC – weighted

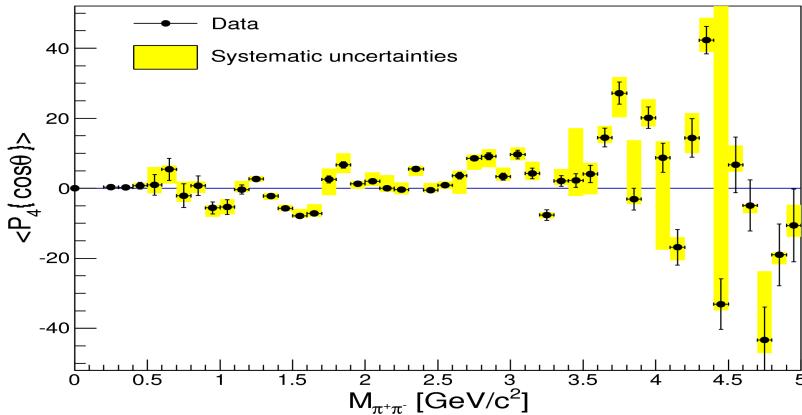


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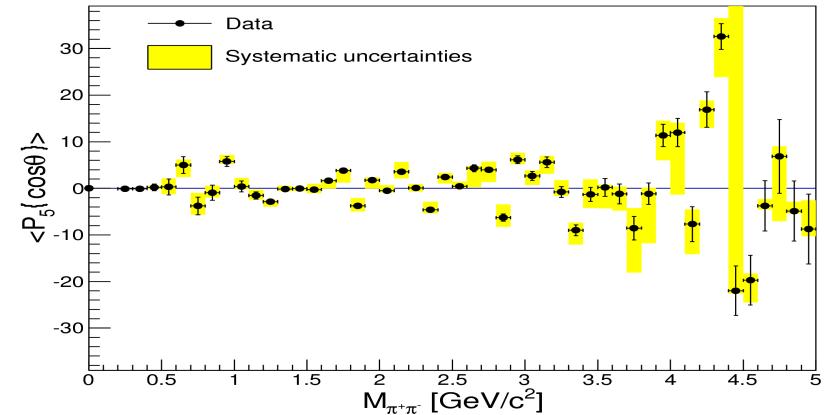


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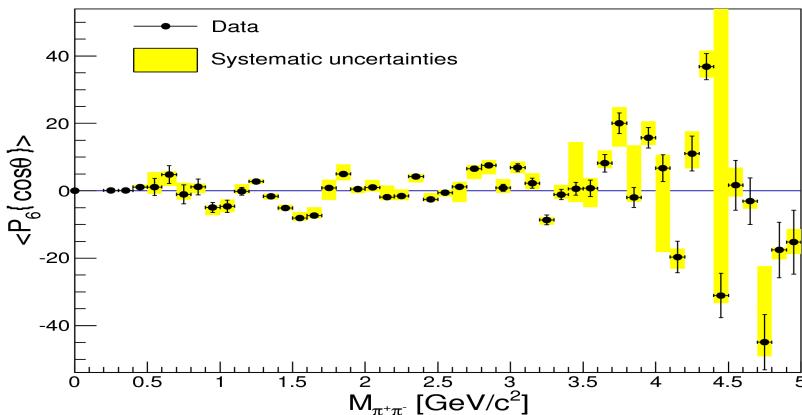
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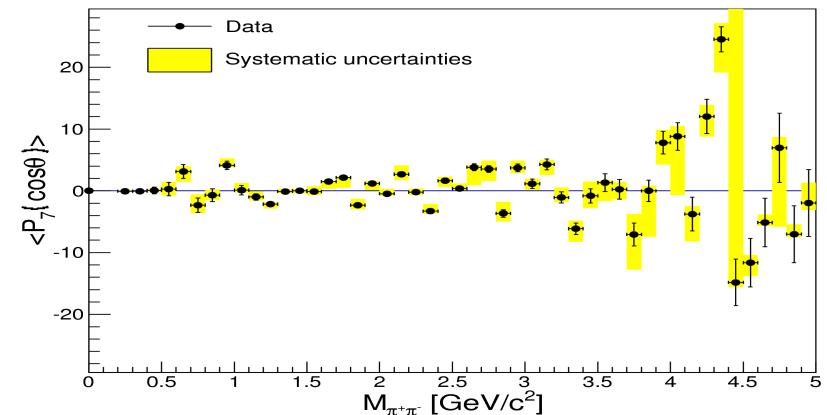
CDF Run II Preliminary



CDF Run II Preliminary

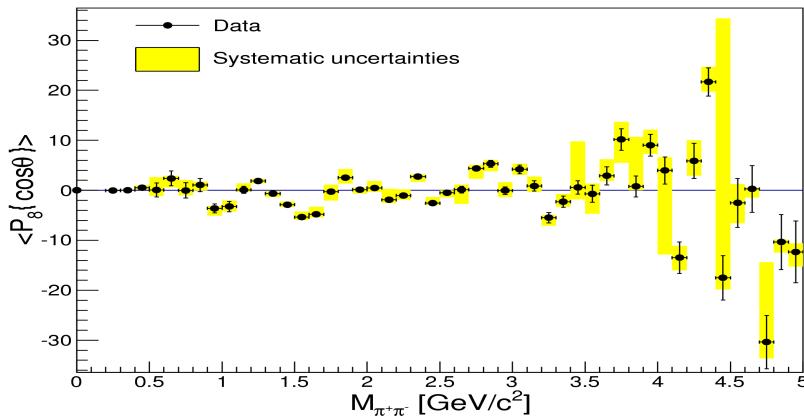


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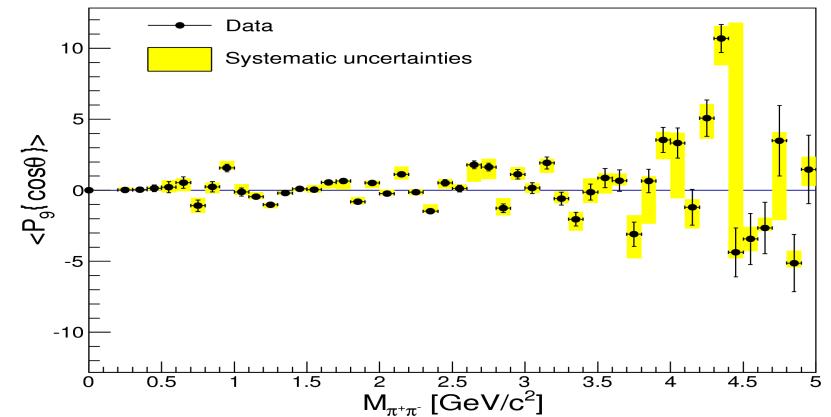


# Data – MC weighted

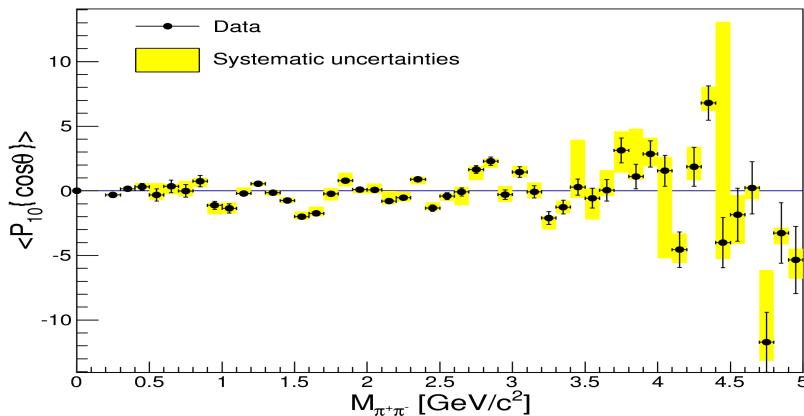
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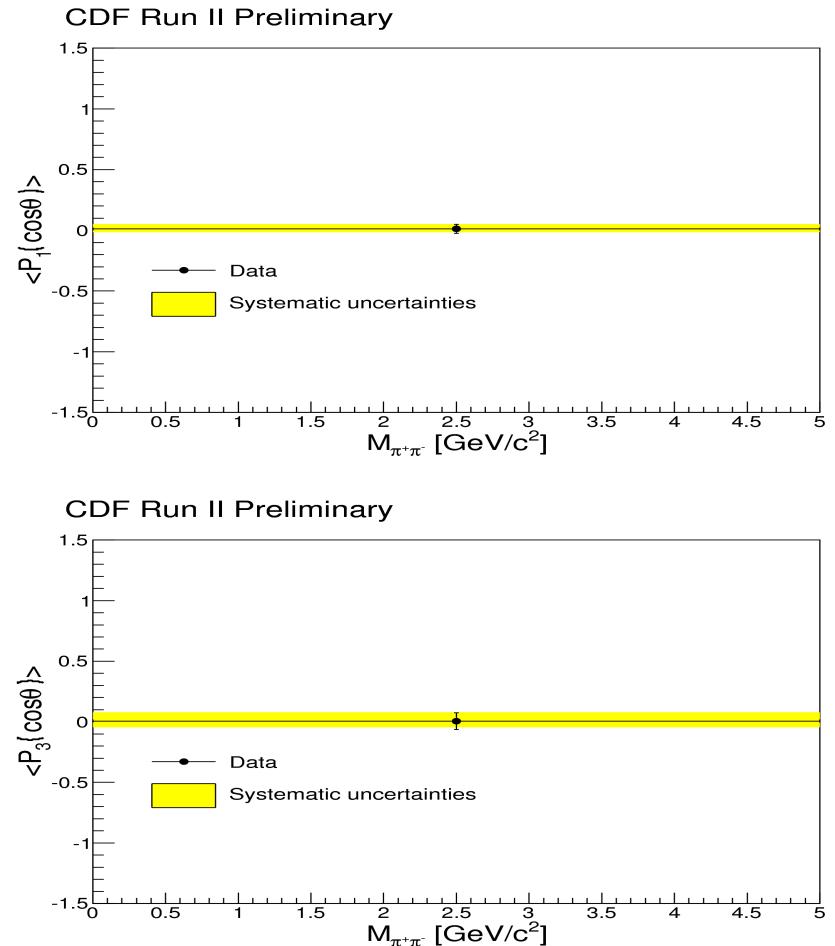
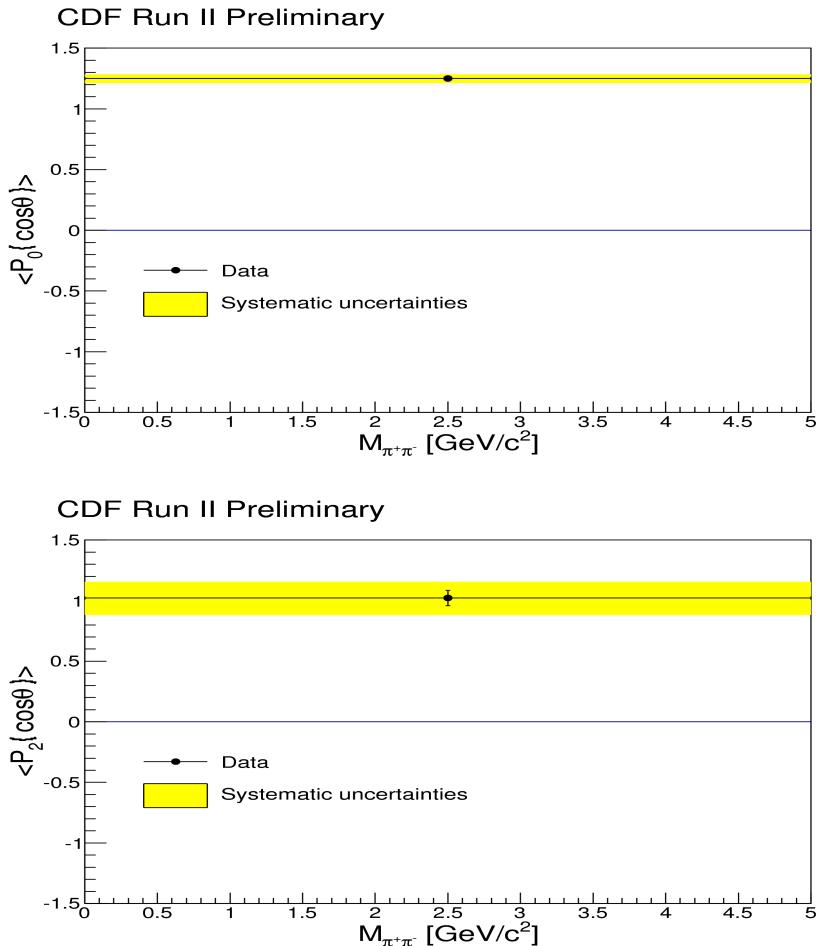
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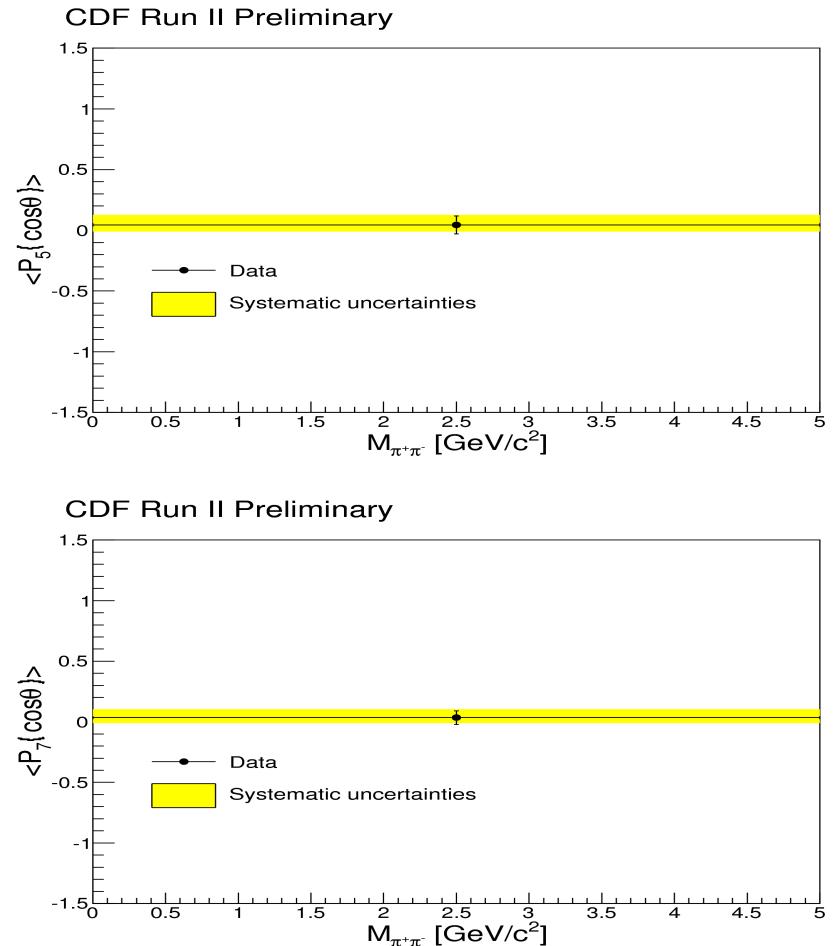
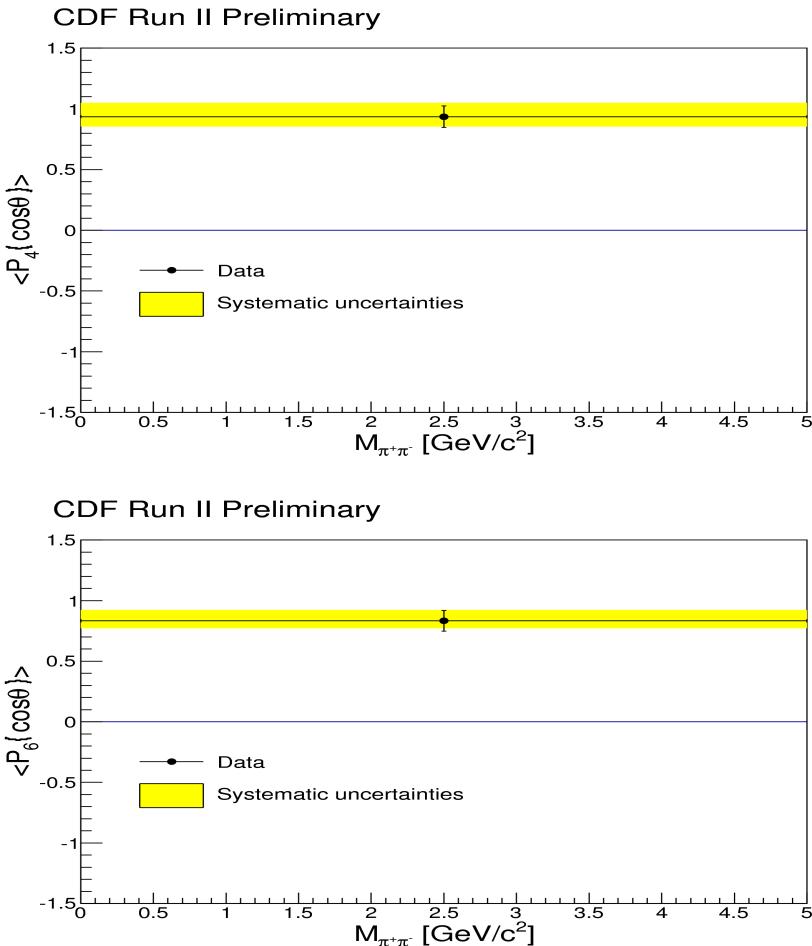
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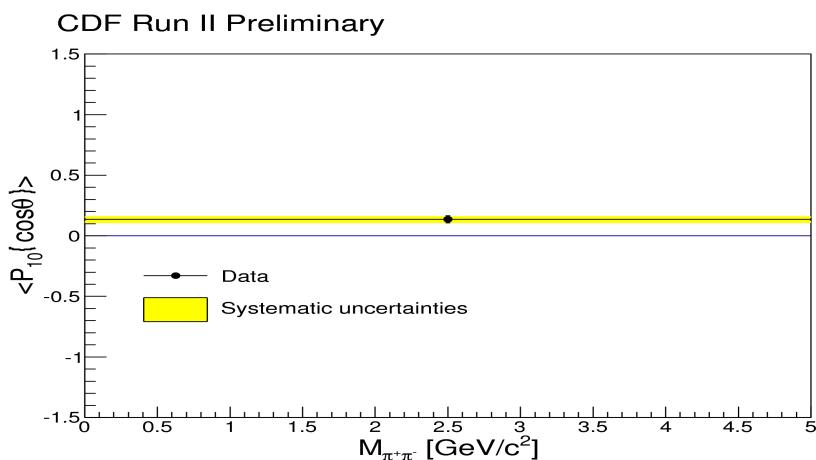
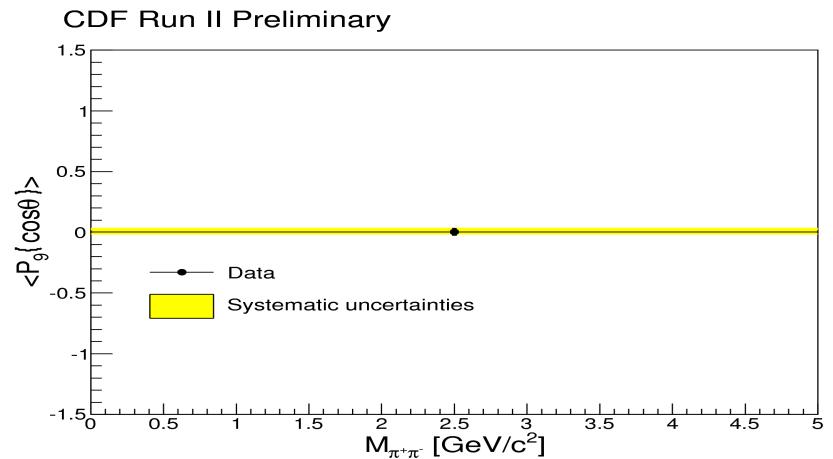
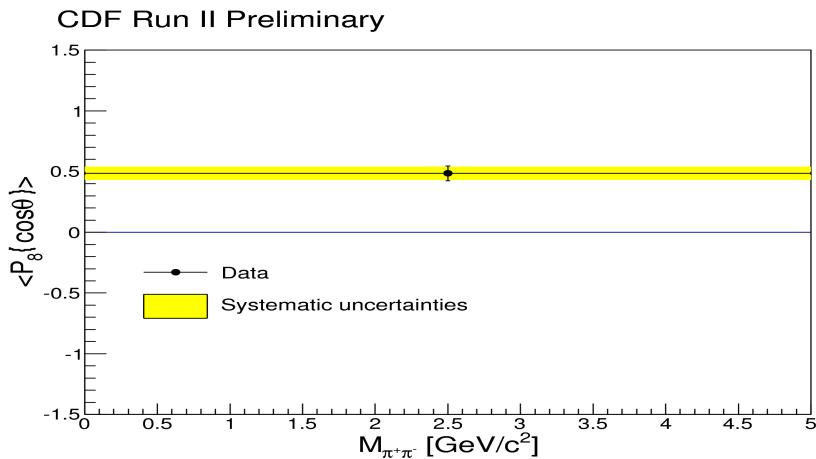
# Single mass bin



# Single mass bin



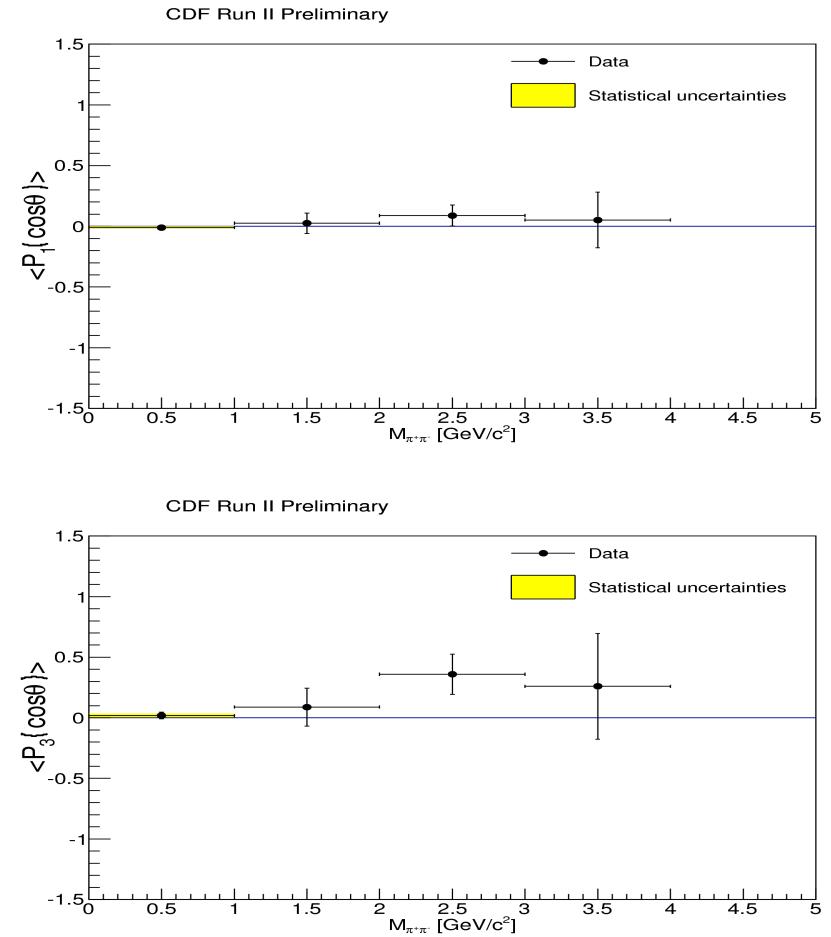
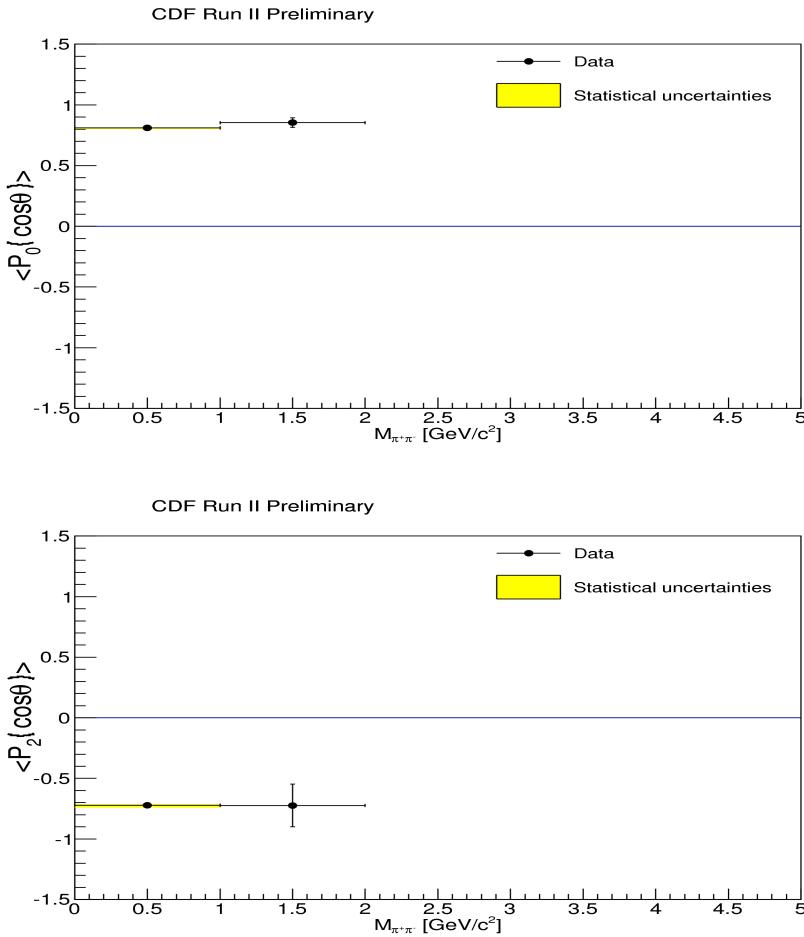
# Single mass bin



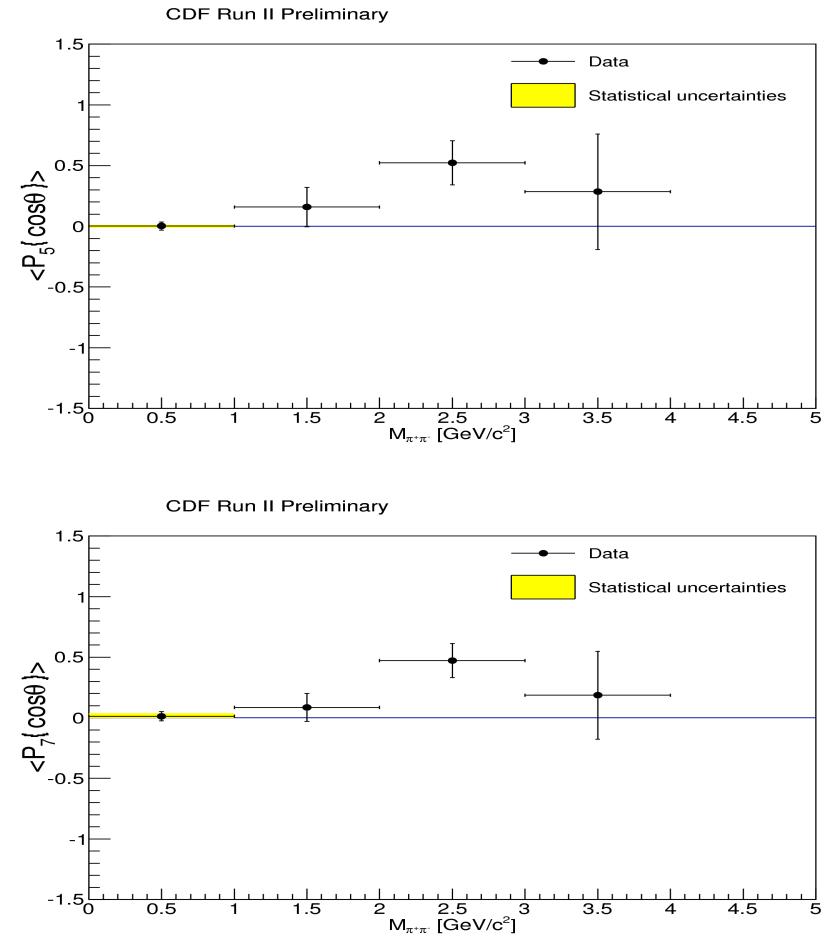
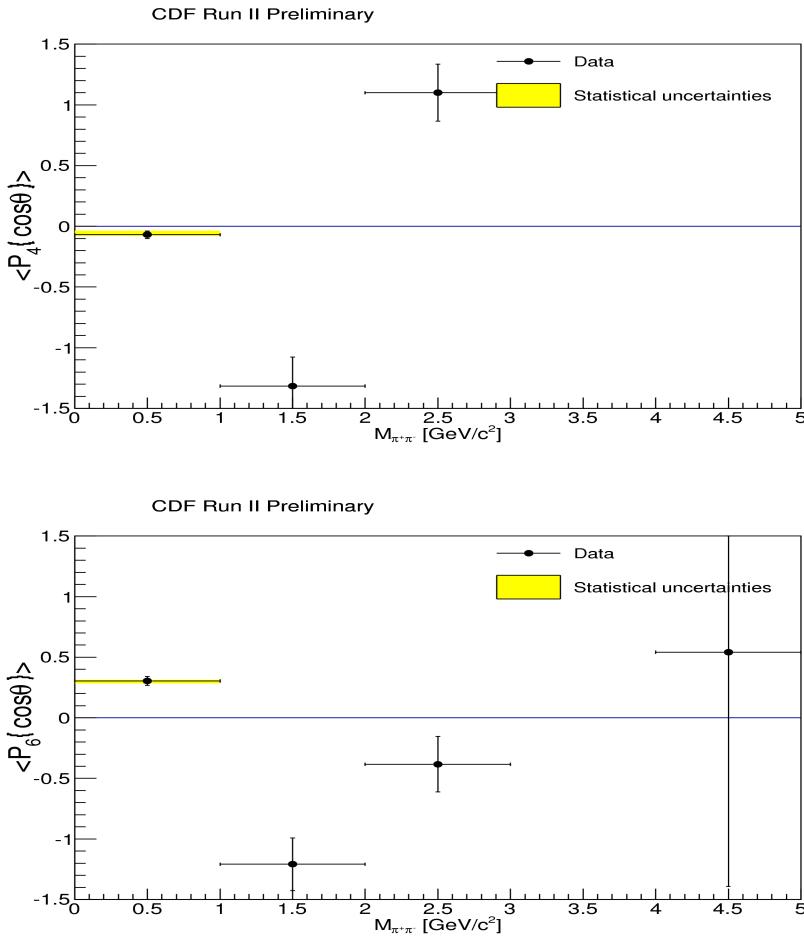
# Thank you

# Backup slides

# 5 mass bins



# 5 mass bins



# 5 mass bins

